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Energy-Scale Dependence of the Lepton-Flavor-Mixing Matrix

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Abstract

We study an energy-scale dependence of the lepton-flavor-mixing matrix in the minimal supersymmetric standard model with the effective dimension-five operators which give the masses of neutrinos. We analyze the renormalization group equations of κ_{ij}s which are coefficients of these effective operators under the approximation to neglect the corrections of $O(\kappa^2)$. As a consequence, we find that all phases in κ do not depend on the energy-scale, and that only $n_g - 1$ (n_g: generation number) real independent parameters in the lepton-flavor-mixing matrix depend on the energy-scale.

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Recent neutrino experiments suggest the existence of the flavor mixing in the lepton sector [1]-[5]. Studies of the lepton-flavor-mixing matrix, which is called [6] Maki-Nakagawa-Sakata (MNS) matrix [7], open a new era of the flavor physics. We can predict the lepton-flavor-violating interactions such as $\mu \to e\gamma$ from the MNS matrix. When we consider that the lepton-flavor-violating interactions are related to the new physics at a high energy-scale, it is important to analyze the energy-scale dependence of the MNS matrix in order to obtain information of the new physics.

In this letter we study the energy-scale dependence of the MNS matrix in the minimal supersymmetric standard model (MSSM) with effective dimension-five operators which give masses of neutrinos. In this model the superpotential of lepton-Higgs interaction terms is

$$\mathcal{W} = {y^{\mathrm{e}}}_{ij} H_{\mathrm{d}} L_i \bar{E}_j - \frac{1}{2}\kappa_{ij}(H_{\mathrm{u}}L_i)(H_{\mathrm{u}}L_j)\,. \tag{1}$$

Here the indices i, j $(= 1 \sim n_g)$ stand for the generation number. L_i and $\bar{E}_i$ are i-th generation lepton doublet and right-handed charged lepton, $H_{\mathrm{u,d}}$ are the Higgs doublets which give Dirac masses to the up- and down-type fermions, respectively. The coefficients matrix (κ) of the effective dimension-five operators which is an $n_g \times n_g$ complex and symmetric matrix, gives the neutrino Majorana mass matrix. When we take the diagonal base of the charged lepton Yukawa coupling y^{e}, κ is diagonalized by the MNS matrix. All the elements of κ are naturally small if they are generated effectively by the new physics at a high energy-scale M. One of the most reliable candidate is so-called see-saw mechanism [8], where the small κ of $O(1/M)$ is generated by the heavy right-handed neutrinos with Majorana masses of $O(M)$.

Now let us consider the renormalization of κ. The wave function renormalization of L_i is given by $L_i^{(0)} = Z_{ij}^{-1/2} L_j$ and that of the Higgs doublet is given by $H_{\mathrm{u}}^{(0)} = Z_H^{-1/2} H_{\mathrm{u}}$. Then the renormalization of κ_{ij} is written as

$$\kappa_{ij}^{(0)} = \left(Z_{ik}^{-1/2} Z_{jl}^{-1/2} Z_H^{-1}\right)\kappa_{kl}\,. \tag{2}$$

Here we adopt the approximation to neglect loop corrections of $O(\kappa^2)$, which are sufficiently small according to the tiny neutrino masses. It corresponds to taking the Feynman diagrams in which κ appears only once. If κ is induced by the see-saw mechanism, this approximation is consistent with neglecting terms of $O(1/M^2)$ in the see-saw mechanism. Under this approximation Z_{ik} becomes diagonal as $Z_{ik} = Z_i \delta_{ik} + O(\kappa^2)$ because there are no lepton-flavor-mixing terms except κ. Therefore eq.(2) becomes simple as

$$\kappa_{ij}^{(0)} = \left(Z_i^{-1/2} Z_j^{-1/2} Z_H^{-1}\right)\kappa_{ij}\,. \tag{3}$$

Equation (3) leads to the RGE of κ_{ij} as

$$\frac{d}{dt}\kappa_{ij} = \left(\gamma_i + \gamma_j + 2\gamma_H\right)\kappa_{ij}\,, \tag{4}$$

where t is the scaling parameter which is related to the renormalization scale μ as $t = \ln \mu$. γ_i and γ_H are defined as

$$\gamma_i = \frac{1}{2}\frac{d}{dt} \ln Z_i \,, \quad \gamma_H = \frac{1}{2}\frac{d}{dt} \ln Z_H \,. \tag{5}$$

By using eq.(4), we can obtain the following two consequences:
(1) *All phases in κ do not depend on the energy-scale.* By using the notation $\kappa_{ij} \equiv |\kappa_{ij}| e^{i\varphi_{ij}}$, eq.(4) is rewritten as

$$\begin{aligned} \frac{d}{dt} \ln \kappa_{ij} &= \frac{d}{dt} \ln |\kappa_{ij}| + i \frac{d}{dt} \varphi_{ij} \\ &= \left(\gamma_i + \gamma_j + 2\gamma_H \right) . \end{aligned} \tag{6}$$

Since γ_i, γ_j and γ_H are real, eq.(6) means

$$\frac{d}{dt} \varphi_{ij} = 0 \,. \tag{7}$$

Therefore we can conclude that the arguments of all the elements of κ are not changed by RG evolutions. We should notice that this result does not necessarily mean that phases of the MNS matrix are independent of the energy-scale as we will see later.
(2) *Only $n_g - 1$ real independent parameters in the MNS matrix depend on the energy-scale.* The combinations of κ's elements,

$$c_{ij}^2 = \frac{\kappa_{ij}^2}{\kappa_{ii}\kappa_{jj}} \,, \tag{8}$$

are the energy-scale independent quantities because

$$\begin{aligned} \frac{d}{dt} \ln \left(\frac{\kappa_{ij}^2}{\kappa_{ii}\kappa_{jj}} \right) &= 2\frac{d}{dt} \ln \kappa_{ij} - \frac{d}{dt} \ln \kappa_{ii} - \frac{d}{dt} \ln \kappa_{jj} \\ &= 2 \left(\gamma_i + \gamma_j + 2\gamma_H \right) - (2\gamma_i + 2\gamma_H) - \left(2\gamma_j + 2\gamma_H \right) \\ &= 0 \,. \end{aligned} \tag{9}$$

Since the off-diagonal elements of the κ_{ij} $(i \neq j)$ are given by

$$\kappa_{ij} = c_{ij} \sqrt{\kappa_{ii}\kappa_{jj}} \quad (i \neq j) \,, \tag{10}$$

their energy-scale dependence can be completely determined by the diagonal elements κ_{ii}. The diagonal elements κ_{ii} can always be taken to be real by rephasing neutrino fields,

and they never become complex by the RGE in eq.(4). The RGE of κ can be written by only n_g equations. The diagonal form of $y^{\rm e}$ is held at any energy-scale because there is no lepton-flavor-violating correction to the RGE of $y^{\rm e}$ up to $O(\kappa)$. Since the overall factor of the κ's elements is nothing to do with the MNS matrix, the energy-scale dependence of the MNS matrix can be determined by $n_g - 1$ real independent parameters.

Let us show an example for the case of three generations. The matrix κ is parameterized as

$$\kappa = \kappa_{33} \begin{pmatrix} r_1 & c_{12}\sqrt{r_1 r_2} & c_{13}\sqrt{r_1} \\ c_{12}\sqrt{r_1 r_2} & r_2 & c_{23}\sqrt{r_2} \\ c_{13}\sqrt{r_1} & c_{23}\sqrt{r_2} & 1 \end{pmatrix} , \tag{11}$$

where

$$r_i \equiv \frac{\kappa_{ii}}{\kappa_{33}} , \quad (i = 1, 2) . \tag{12}$$

The complex parameters, c_{ij} are energy-scale independent. There are nine degrees of freedom, which are three complex constants c_{ij} $(i \neq j)$ and three energy-scale dependent real parameters r_1, r_2 and κ_{33}. Since $y^{\rm e}$ has diagonal form at any energy-scale and κ_{33} is nothing to do with the MNS matrix, only two parameters, r_1 and r_2, are energy-scale dependent parameters in the MNS matrix.

Here we roughly estimate the energy-scale dependence of the r_i in eq.(12) by using the one-loop RGEs in the MSSM [9, 10]. We can easily show the RGE of r_i is given by

$$\frac{d}{dt} \ln r_i = \frac{d}{dt} \ln \frac{\kappa_{ii}}{\kappa_{33}} = -\frac{1}{8\pi^2} \left(y_\tau^2 - y_i^2 \right) , \quad (i = 1, 2) , \tag{13}$$

where y_τ and y_i are Yukawa couplings of τ and i-th generation charged lepton, respectively. Neglecting the energy-scale dependence of y_τ, the magnitude of the right-hand-side in eq.(13) is roughly given by $y_\tau^2(m_Z)/8\pi^2 = O(10^{-6})/\cos^2\beta$, where m_Z stands for the weak scale and $\tan\beta = \langle H_{\rm u} \rangle / \langle H_{\rm d} \rangle$. It means that r_is are not sensitive to the energy-scale. We should stress here that this fact does not necessarily result in the tiny energy dependence of the MNS matrix. We can explicitly see the significant RGE corrections of the MNS matrix in some situations [9, 10]. In ref. [10], the drastic change of the MNS matrix by the RGE was obtained when neutrinos of the second and third generations have masses of $O(\text{eV})$ with $\delta m_{23}^2 \simeq 3 \times 10^{-3}$ (eV^2) [3]. This situation corresponds to the case of $r_1 \sim |c_{12}| \sim 0$, $r_2 \sim 1$, and $|c_{23}| \ll 1$ in eq.(11), where the slight change of r_2 induces the maximal mixing of the second and third generations in the MNS matrix.

In this letter we studied an energy-scale dependence of the MNS matrix in the MSSM with the effective dimension-five operator. The coefficient of the dimension-five operator

κ is small enough to neglect corrections of $O(\kappa^2)$ in RGEs. Under this approximation we found that all phases in κ do not depend on the energy-scale, and that only $n_g - 1$ real independent parameters in the MNS matrix depend on the energy-scale. Our consequences imply that there must be $(n_g - 1)^2 = n_g(n_g - 1) - (n_g - 1)$ scale independent relations among the MNS matrix elements because the MNS matrix generally has $n_g(n_g - 1)$ real independent parameters when neutrinos are Majorana fermions. These results can be helpful for the lepton flavor physics and search for the new physics at the high energy-scales.

Finally we discuss the possibility to obtain the same consequences in other models with the effective dimension-five operators. The supersymmetry (SUSY) is needed to obtain our consequences. Moreover, it was necessary that the model did not have lepton-flavor-violating terms and non-renomalizable terms except the effective dimension-five operators. Thus we can obtain the same consequences in the SUSY models which have these properties, *e.g.* the Next-MSSM. On the other hand we cannot directly apply our analysis to the standard model or non-SUSY models because non-zero vertex renormalization generates an additional term in the right-hand-side of eq.(4), which is generally not real. Nevertheless, we can explicitly show that this term is real at one-loop level [9]. Therefore we can obtain the same consequences in the standard model at one-loop level.

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